A BAYESIAN ANALYSIS OF HUME'S ARGUMENT CONCERNING MIRACLES

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There have recently appeared in *The Philosophical Quarterly* two very interesting papers attempting to give a Bayesian analysis of Hume's argument concerning miracles.¹ In the present paper we shall attempt to give a shorter and simpler treatment of the question, and one which, we think, makes clearer the difference between Hume and his contemporary critic Price. Our analysis is based on a general Bayesian account of testimony given by Dawid.² In the course of expounding it, we shall explain the points where it differs from the earlier analyses of Sobel and Owen.

1. HUME'S ARGUMENT AND PRICE'S REPLY

Hume's argument concerning miracles is familiar to most philosophers, but we will begin by describing it briefly. Hume himself summarizes the argument as follows:

... no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the fact, which it endeavors to establish; and even in that case there is a mutual destruction of arguments, and the superior only gives us an assurance suitable to that degree of force, which remains, after deducting the inferior.³

For Hume a miracle is a violation of the laws of nature, and a law of nature is something for which there is very strong empirical evidence. As he says:

A miracle is a violation of the laws of nature; and as a firm and

unalterable experience has established these laws, the proof against a miracle, from the very nature of the fact, is as entire as any argument from experience can possibly be imagined."

A miracle can thus only be established if there is very strong evidence, provided by testimony, in its favour. Yet it is not miraculous, or even unusual, for witnesses to lie. Besides, mankind has a strong propensity to the marvellous, and there are many examples of faked miracles. As Hume says:

The many instances of forged miracles, and prophecies, and supernatural events, which, in all ages, have either been detected by contrary evidence, or which detect themselves by their absurdity, prove sufficiently the strong propensity of mankind to the extraordinary and the marvellous, and ought reasonably to beget a suspicion against all relations of this kind."

Hume, characteristically, is particularly sceptical about miracles cited in support of some religion, observing: "But what greater temptation than to appear a missionary, a prophet, an ambassador from heaven? ... who ever scruples to make use of pious frauds, in support of so holy and meritorious a cause?"

Hume, as we have seen, gives the argument in informal terms. In this paper we want to attempt a Bayesian formulation of the argument using the mathematical theory of probability. This project is very far from being a novel one, and was in fact closely connected with the origins of Bayesianism. Thomas Bayes' fundamental paper of 1763 was presented to the Royal Society after Bayes' death by his friend Richard Price, who wrote an introduction and an appendix. In his (1987), Gillies shows that there were close links between Price and Hume. Price knew Hume personally. His introduction and appendix to Bayes' (1763) essay were attempts to use Bayes' theorem to answer Hume's doubts about induction. Moreover, the last of Price's Four Dissertations, published in 1767, was entitled The Importance of Christianity, the Nature of Historical Evidence, and Miracles. Here Price uses Bayesian considerations in an attempt to answer Hume's argument about miracles. In his (1987) Gillies has given an account of

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4 Hume (1748), § 90, p. 114.
5 Hume (1748), § 93, p. 118.
6 Hume (1748), § 97, p. 125.
Price's discussion of miracles. Here we will content ourselves with mentioning what seem to us to be Price's two most important points.

First of all Price uses his Bayesianism to argue that evidence never establishes a law of nature with certainty, but only with some probability less than one. There is thus a possibility that the evidence of testimony for the miracle will outweigh the evidence for the law of nature.

Second, Price has an ingenious argument designed to show that human testimony of no extraordinary character is sometimes taken as establishing an event of very low probability. Consider a lottery in which a six-figure number is drawn at random. The probability of any particular number being chosen is vanishingly small ($10^{-6}$). Yet when the winning number is given in the evening paper, we usually believe the report without checking the matter further. Price gave the example as follows:

The improbability of drawing a lottery in any particular assigned manner, independently of the evidence of testimony, or of our own sense, acquainting us that it has been drawn in that manner is such as exceeds all conception. And yet the testimony of a newspaper, or of any common man, is sufficient to put us out of doubt about it.\(^{10}\)

Price's argument is an interesting one, but not entirely convincing. We may well ask whether the lottery case is really analogous to that of miracles. In a lottery, a particular event (the drawing of a particular number) is indeed very improbable, but the occurrence of an event of that type (the drawing of some six-figure number) is not only not improbable, but even virtually certain. In the case of a miracle, however, not only is the particular event (a particular man levitating at a particular time and place) improbable, but so also is any event of that type (any man levitating at any time and place).

The question could perhaps be resolved by pursuing Price's Bayesian line of thought, and giving a more precise mathematical analysis of the effects of testimony. It might then be possible to calculate whether the relevant probabilities in the miracles' case are the same as in the lottery case, or whether the two cases are significantly different. Although Price was a highly skilled mathematician, he never attempted such an analysis, and this is what we intend to do in the remainder of this paper.

2. A BAYESIAN ANALYSIS OF TESTIMONY

Let \( A \) be an event, and \( \bar{A} \) the event that \( A \) did not occur. Let \( a \) be the testimony of a witness or group of witnesses \( W \) that \( A \) did occur.

By Bayes’ Theorem:

\[
(1) \quad Pr(A|a) = \frac{Pr(a|A) Pr(A)}{Pr(a|A) Pr(A) + Pr(a|\bar{A}) Pr(\bar{A})}
\]

Here \( Pr(A) \) is the prior probability of \( A \). It must be understood not as a priori in an absolute sense, but as relative to some assumed background knowledge \( K \), so that \( Pr(A) = Pr(A|K) \), and similarly, \( Pr(A|a) = Pr(A|a \& K) \). For convenience of mathematical calculation, we shall not usually mention \( K \) explicitly in our formulas. However, the relativity to background knowledge should not be forgotten, for, as we shall see, an assessment of Hume’s argument concerning miracles depends crucially on the nature of the background knowledge \( K \).

Writing \( Pr(A) = p \), we have, since \( Pr(\bar{A}) = 1 - p \),

\[
(2) \quad Pr(A|a) = \frac{p Pr(a|A)}{p Pr(a|A) + (1 - p) Pr(a|\bar{A})}
\]

This is the formula we shall use in our analysis of the lottery case and the miracles case.\(^1\)

3. ANALYSIS OF (i) THE LOTTERY CASE, AND (ii) THE MIRACLES CASE

Before proceeding to detailed analyses of the two cases, it will, I think, be helpful to make some general remarks about formula (2). This formula enables us to calculate \( Pr(A|a) \), that is the probability of event \( A \) having occurred given that \( W \) says it occurred, and, of course, some background

\(^1\) This analysis is based on the work of Dawid, and we have used the same notation as Dawid, cf. his (1987) p. 93.

\(^2\) Owen in his treatment of the question considers the case where \( Pr(a|\bar{A}) = 1 - Pr(a|A) \) and setting \( Pr(a|A) = t \), obtains from (2) the formula

\[
\frac{pt}{pt + (1 - p)(1 - t)}
\]

Cf. Owen (1987) p. 191. This is the formula which he uses in his Bayesian analysis of Hume. However, in general \( Pr(a|\bar{A}) \neq 1 - Pr(a|A) \), and, in particular, an analysis of the lottery case depends on giving a value to \( Pr(a|\bar{A}) \) which differs considerably from \( 1 - Pr(a|A) \). Thus Owen’s mathematical treatment seems to us too simplified to give an adequate account of the situation. However, Owen’s paper contains some valuable philosophical points, some of which will be mentioned below.
knowledge. In order to calculate Pr (A|a), we have to assess three other probabilities, namely:

(α) p the prior probability of the event having occurred, that is, its probability relative to background knowledge alone before we have the testimony provided by W;

(β) Pr (a|A). This is the probability that W will say that A occurred, given that A did occur. Our estimate of this probability will depend on our judgement of the reliability of W;

(γ) Pr (a|Ā). This is the probability that W will say that A occurred, given that A did not occur. Here again our estimate of this probability will depend on our judgement of the reliability of W, as well as on other relevant factors.

(i) Lottery case

By the way the example was set up, we have immediately that p = prior probability of a particular number coming up = 10^{-6}. Next consider Pr (a|A). This is the probability of the paper’s printing a particular number, given that that particular number really was the result of the lottery. Informally we are here concerned with the probability of the newspaper’s getting it right, as opposed to making a mistake, or, conceivably, a deliberate deception. If the newspaper’s staff take care to avoid such mistakes and deceptions, and if the newspaper has a good record in this respect, we can reasonably estimate Pr (a|A) = 0.8.

We are now left with what turns out to be the most crucial part of the analysis – the estimate of Pr (a|Ā). Let I be the event that the newspaper prints an incorrect number i.e. a number different from the one which was the actual result of the lottery. Then we have

\[ a & Ā = a & I \]

so

\[ Pr (a|Ā) Pr (Ā) = Pr (a|I) Pr (I) \]

\[ Pr (a|Ā) = \frac{Pr (a|I) Pr (I)}{1 - p} \]

Here p = 10^{-6}.

Suppose our earlier estimate of Pr (a|A) holds for any result A of the lottery. Then Pr (I) = 1 - 0.8 = 0.2. We are left with the task of estimating Pr (a|I). Knowing only that the newspaper will print an incorrect
result (but not knowing the true result) we can not rule out any of the \(10^6\) numbers as that which will appear. So, if we discount possibilities such as the newspaper having favourite numbers it likes to print, or typographical errors favouring repeated digits – which perhaps should not be discounted too easily – we should take \(\Pr (a|I) = 1/10^6\).

So

\[
\Pr (a|\bar{A}) = \frac{0.2}{10^6 (1 - 10^{-6})}
\]

Substituting these values in (2) above, we get

\[
\Pr (A|a) = \frac{0.8 \times 10^{-6}}{0.8 \times 10^{-6} + 0.2 \times 10^{-6}} = 0.8
\]

So here the Bayesian analysis agrees with common sense. If we judge the evening paper to be pretty reliable, we can assign a high probability to its report of the winning number being correct, even though the prior probability of that particular number being the winning number is very low indeed. But does the same hold in the case of miracles, as Price claims? This is what we must consider next.

(ii) Miracles case

We shall assume that a miracle has, relative to our background knowledge, a very low, but still finite probability.\(^{11}\) To bring out the parallel with the lottery case, we shall in fact again take \(p = 10^{-6}\). To make the case as favourable as possible to the establishment of the miracle, let us suppose that \(W\) consists of a group of very sober, reliable and trustworthy individuals, and that they claimed to observe the miracle in circumstances which would make deception very difficult (a public spot, the clear light of a bright sunny day, etc.). We might then take \(\Pr (a|A) = 0.99\).

\(^{11}\) In this case \(1 - \Pr (a|A) = 0.2\) and so is very different from \(\Pr (a|\bar{A})\), as was claimed in note 12 above.

\(^{12}\) This is the point on which our treatment of the question differs from that of Sobel. Sobel uses the ideas of non-standard analysis, and takes a miracle to be an event whose probability is less than some positive infinitesimal, cf. Sobel 1987, p. 174. This does not seem to us correct from a Bayesian point of view, since, as Price pointed out, however great the quantity of evidence in favour of a law of nature, the law itself, and predictions based on the law, will still be allotted probabilities less than 1 on the evidence by a Bayesian. Owen (1987, p. 189, fn. 6) criticizes Sobel, in our view quite correctly, on this point. It should be added, however, that Sobel’s analysis of the lottery case, (1987, pp. 179–80), is substantially the same as that just given.
Note that this is much higher than in our analysis of the lottery case.

We have finally to evaluate $Pr(A|\bar{A})$, that is, the probability that our group of very sober, reliable and trustworthy individuals reported witnessing a miracle when no such thing actually occurred. Clearly this probability is low, but how low? Here we must remember Hume's points about the frequency of faked miracles, and of mankind's propensity to the marvellous. We must also bear in mind the possibility of deception and the skills of professional magicians. Then again the influence of strong religious feeling could manifest itself either in pious fraud (as Hume suggests), or perhaps in some form of mass hallucination. Given all these considerations, it would seem unreasonable to us to take $Pr(A|\bar{A})$ much lower than $10^{-3}$.

Substituting these values, we get

$$Pr(A|a) = \frac{0.99 \times 10^{-6}}{0.99 \times 10^{-6} + 10^{-3} (1 - 10^{-6})}$$

$$= \frac{0.99 \times 10^{-3}}{0.99 \times 10^{-3} + (1 - 10^{-6})}$$

$$\approx 10^{-3}$$

So it emerges, contrary to Price, that, on a Bayesian analysis, the lottery case and the miracles case are different. In the lottery case, the reported result has a very low prior probability, but the probability of a particular wrong number being reported is also very low. In the calculations these two low probabilities cancel out enabling us to assign a reasonably high probability to the reported result being correct. In the miracles case, the prior probability of the miracle is also very low, but there is no other very low probability in the formula which cancels this out. Even making the most favourable possible assumptions regarding the reliability of the testimony, the probability of the miracle given that testimony is still too low to make the miracle believable. It looks as if we must conclude that a Bayesian analysis shows Hume's argument concerning miracles to be correct. Such a conclusion, however, needs to be modified in the light of some considerations which we will put forward in the last section of this paper.
4. AN ASSESSMENT OF HUME’S ARGUMENT CONCERNING MIRACLES

Assuming the above Bayesian analysis, was Hume correct in thinking that no amount of testimony can establish that a miracle has occurred? Let \( A \) be the event of the miracle occurring, and \( K \) our background knowledge. If we assume, as we did above, that \( \Pr(A|K) \) is of the order of \( 10^{-6} \), then Hume’s argument is certainly correct. But now suppose we take \( \Pr(A|K) \) as \( 10^{-3} \), but evaluate \( \Pr(a|A) \) and \( \Pr(a|\bar{A}) \) as before. We then have

\[
\Pr(A|a) = \frac{0.99 \times 10^{-3}}{0.99 \times 10^{-3} + 10^{-3} (1 - 10^{-3})} \triangleq 0.99
\]

Thus if we are prepared to raise \( \Pr(A|K) \) to \( 10^{-3} \), very good testimony might, after all, make it reasonably probable that a miracle had occurred. It is clear that a lot depends on how we evaluate the probability of a miracle relative to our background knowledge \( (K) \).

Suppose, to begin with, that we allow into \( K \) only empirically based scientific knowledge. Then surely \( \Pr(A|K) \) must be very low indeed, since \( A \), by definition, is something highly unexpected in the light of \( K \). This shows that someone who has a strictly scientific view of the world can never be convinced of the truth of religion by testimony in favour of miracles.

But next suppose that \( K \) is taken to include some religious knowledge \( (K', \text{ say}) \) based perhaps on revelation or metaphysical considerations. Relative to \( K' \), a miracle is still unlikely, but not so unlikely. Someone believing in \( K' \) might see it as distinctly possible for God to perform a miracle in order to convey some important message to humanity. It turns out then that Hume’s argument might be circumvented by someone who allowed the possibility of religious knowledge as distinct from empirically based scientific knowledge.

This conclusion agrees, broadly speaking, with that of Swinburne in his book on miracles. Swinburne writes:

With one Weltanschauung (‘world-view’) one rightly does not ask much in the way of detailed historical evidence for a miracle since miracles are the kind of events which one expects to occur in many or certain specific circumstances. The testimony of one witness to an occurrence of the kind of miracle which in its circumstances
one would expect to happen should be sufficient to carry conviction, just as we accept the testimony of one witness to a claim that when he let go of a book which he was holding it fell to the ground. With another Weltanschauung one rightly asks for a large amount of historical evidence, because of one's general conviction that the world is a certain sort of world, a world without a god and so a world in which miracles do not happen. Which Weltanschauung is right is a matter of long argument.\textsuperscript{15}

Owen too makes the following, quite similar, observation:

If one \textit{already} believed in the God of the Christian religion, then if the sort of evidence envisaged became available, it might be rational to treat the violation as a result of God's volition. But if one was not already a believer, then even if such evidence obtained, one would still have no good reason to change one's mind.\textsuperscript{16}

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\textsuperscript{15} Swinburne, R. (1970) \textit{The Concept of Miracle} (London: Macmillan), Ch. 6, p. 71.

\textsuperscript{16} Owen 1987, p. 201.